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# Predicting unsaturated hydraulic conductivity of soil based on some basic soil properties

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## Abstract

Soil hydraulic conductivity is a crucial parameter in modeling flow process in soils and deciding water management. In this study, by combining the non-similar media concept (NSMC) to the one-parameter model of Brooks and Corey, a new NSMC-based model for estimating unsaturated hydraulic conductivity of various soils was presented. The main inputs are soil bulk density, particle-size distribution, soil water retention characteristic and saturated hydraulic conductivity of soil. The results indicated that the NSMC-based model could generally more accurately predict unsaturated hydraulic conductivity of soils, as compared to four one-parameter models and van Genuchten–Mualem model. This study, by introducing NSMC, provided a new way to incorporate soil physical heterogeneity into soil hydraulic simulation, and hence NSMC-based approach is expected to improve efficiency of the existing models in the simulation of soil water flow.

*Keywords:* Unsaturated hydraulic conductivity; Non-similar media concept; One-parameter model

## 1. Introduction

Of soil hydraulic properties, hydraulic conductivity, including saturated hydraulic conductivity, $K_s$, and unsaturated hydraulic conductivity, $K_{us}(\theta)$ or $K_{us}(\psi)$, is a crucial parameter. Numerous models have been presented since the beginning of this century, and have provided many useful insights into the water flow phenomena of soil. In view that direct determination of hydraulic conductivity, especially $K_{us}(\theta)$ or $K_{us}(\psi)$, is difficult, time consuming and expensive, many efforts have been made to indirectly predict this parameter from soil variables routinely measured in the laboratory or in the field (Vereecken et al., 1990; van Genuchten and Leij, 1992). For $K_{us}(\theta)$ or $K_{us}(\psi)$, many methods (Brooks and Corey, 1964; Mualem, 1976; Alexander and Skaggs, 1986; Poulsen et al., 1998) have been developed to approximate it from the soil water retention characteristic, $\psi$–$\theta$. At present, these kinds of indirect methods mainly have two types, i.e. empirical one-parameter models and such statistical prediction models as van Genuchten–Mualem model (van Genuchten and Leij, 1992). These methods are generally based on capillary tube models of water flow through soil pores. However, most of the work under this assumption is confounded

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**Nomenclature**

*Hydraulic properties of soil*

| | |
|---|---|
| $C$ | percentage content of clay particle (<2 μm) of soil (%) |
| $d_g$ | mean geometric diameter of solid particles (mm) |
| $D_{hyd}$ | mean geometrical diameter of capillary tubes in the grouped flow domain (mm) |
| $K_s$, $K_{s0}$ | saturated hydraulic conductivity of soils investigated and referenced, respectively (cm s$^{-1}$) |
| $K_{us}(\theta)$ | unsaturated hydraulic conductivity of soil (cm s$^{-1}$) |
| $M$ | solid mass per unit volume soil (Mg) |
| $n$ | shape parameter of van Genuchten model |
| $S_e$ | effective degree of saturation of soil water |
| $V_e$ | equivalent unsaturated flow-active pore volume (m$^3$) |
| $V_s$, $V_w$ | volumes of soil solid and liquid phases per unit volume soil (m$^3$) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | shape parameter of van Genuchten model |
| $\beta$ | parameter equivalent to residual water content of soil medium (dimensionless) |
| $\gamma$, $\omega$ | empirical constants related to pore conductivity |
| $\varepsilon$ | regulation parameter (dimensionless) |
| $\theta$ | soil water content (m$^3$ m$^{-3}$) |
| $\theta_r$, $\theta_s$ | residual and saturated water contents of soil, respectively (m$^3$ m$^{-3}$) |
| $\rho_b$, $\rho_{b0}$ | bulk density of soils investigated and referenced, respectively (Mg m$^{-3}$) |
| $\rho_s$, $\rho_{s0}$ | particle densities of soils investigated and referenced, respectively (Mg m$^{-3}$) |
| $\tau_{max}$, $\tau_0$ | the maximum and minimum values of $\tau$, respectively (dimensionless) |
| $\phi_e$ | saturated flow-active porosity (dimensionless) |
| $\psi$ | soil water potential (cm $H_2O$) |
| $\psi_e$ | air-entry potential (cm $H_2O$) |

*Statistical terminology*

| | |
|---|---|
| $K_{estimated}$ | unsaturated hydraulic conductivity estimated by means of the models (cm s$^{-1}$) |
| $K_{measured}$ | unsaturated hydraulic conductivity measured (cm s$^{-1}$) |
| $N$ | number of hydraulic conductivity |

by the complex pore characteristic which usually results in non-homogeneous velocity field of water flow and in turn often induces large spatial variability of soil hydraulic properties (Mohanty et al., 1994). Therefore, to improve the existing models, combination of micro- and macro-heterogeneity in the mathematical framework is greatly necessary.

Recently, an attractive theory of physically combining soil non-homogeneity into models of soil hydraulic conductivity, referred to as non-similar media concept (NSMC), was presented by Miyazaki (1996). Based on this new concept, parameters characterizing soil physical variability that are available are expected to incorporate into the existing models. Moreover, we note that the one-parameter models for estimating $K_{us}(\theta)$ involve different empirical constants, which are obtained through correlating water retention curve to hydraulic conductivity curve of specific soil samples. However, we think that the empirical parameters should be related to some basic physical properties of soil samples, and more universal model, which has high prediction quality, should be developed. Therefore, objectives of this work were mainly to develop a new model for predicting $K_{us}(\theta)$, by incorporating the NSMC-based $K_s$ scaling model (Miyazaki, 1996) to the one-parameter $K_{us}(\theta)$ model of Brooks and Corey (1964), and simultaneously to compare the results with those estimated by some other models.

## 2. Theory

### 2.1. van Genuchten–Mualem model and one-parameter models for predicting $K_{us}(\theta)$

One of the most popular analytical functions for predicting $K_{us}(\theta)$ is the van Genuchten–Mualem (1992) model (hereafter called VG–M model), like

$$K_{us}(\theta) = K_s S_e^{0.5}[1 - (1 - S_e^{n/(n-1)})^{(1-1/n)}]^2 \qquad (1)$$

which is a combination of the water retention model of van Genuchten (1980)

$$S_e = [1 + (\alpha|\psi|)^n]^{1/n-1} \tag{2}$$

to the general integral conductivity model of Mualem (1976). In the above model, $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$ is the effective degree of saturation, $\alpha$ and $n$ are shape parameters. $\theta_r$ and $\theta_s$ are residual and saturated water contents of soil, respectively, and $\psi$ denotes soil water potential (cm $H_2O$).

Moreover, many one-parameter models for estimating unsaturated hydraulic conductivity of soils were developed (Brooks and Corey, 1964; Mualem, 1976; Alexander and Skaggs, 1986; Poulsen et al., 1998), due to their simplicity in parameterization (Poulsen et al., 1998). Among them, there are two widely used one-parameter models. One is the model of Brooks and Corey (1964) (hereafter called BC model), like

$$K_{us}(\theta) = K_s\left(\frac{\theta}{\theta_s}\right)^{2b+3} \tag{3}$$

Note that Eq. (3) is a little different from the original version of the model of Brooks and Corey (1964), as $S_e$ in the original equation is replaced by $\theta/\theta_s$ (Campbell, 1974). Another is the model of Alexander and Skaggs (1986) (hereafter called A–S model), like

$$K_{us}(\theta) = K_s\left(\frac{\theta}{\theta_s}\right)^{b+3} \tag{4}$$

In Eqs. (3) and (4), $b$ is the parameter fitted by the water retention model of Brooks and Corey (1964) by assuming $\theta_r = 0$, like

$$\frac{\psi}{\psi_e} = \left(\frac{\theta_s}{\theta}\right)^b \tag{5}$$

where $\psi_e$ is the air-entry potential (cm $H_2O$). Poulsen et al. (1998) also proposed two models. One is the SLC (single log conductivity) model obtained through optimizing the empirical constant of the model of Libardi et al. (1980), using conductivity and retention data of 40 sieved soils, given as

$$K_{us}(\theta) = K_s \exp\left[\left(\frac{\theta}{\theta_s} - 1\right)\left(\frac{0.46b}{\theta_s} + 2.6\right)\right] \tag{6}$$

Another is the DLC (double log conductivity) model, representing a modification of Campbell model, with a form as

$$K_{us}(\theta) = K_s\left(\frac{\theta}{\theta_s}\right)^{1.5b+10/3} \tag{7}$$

### 2.2. NSMC-based $K_{us}(\theta)$ model

Miyazaki (1996), theoretically based on a NSMC, presented a scaling model of $K_s$, like

$$\frac{K_s}{K_{s0}} = \left[\frac{(\tau\rho_s/\rho_b)^{1/3} - 1}{(\tau\rho_{s0}/\rho_{b0})^{1/3} - 1}\right]^2 \tag{8}$$

where $K_{s0}$ is the measured saturated hydraulic conductivity of a reference sample with bulk density $\rho_{b0}$, $K_s$ is the estimated saturated hydraulic conductivity of soil sample with a bulk density of $\rho_b$, and $\rho_s$ and $\rho_{s0}$ are particle densities of soils investigated and referenced, respectively. The shape factor $\tau$ is a ratio of the solid phase volume to the total soil volume. Its value is 1.0 for a cube and $\pi/6$ for a sphere. However, for soil, it is difficult to estimate the value of $\tau$. Based on the assumptions that saturated flow is a kind of path flow among which some small particles exist, and the paths are homogeneous in flow velocity, while each of them is composed of many capillary tubes between which flow velocity is different, Zhuang et al. (2000b) proposed an empirical relationship between the shape factor, $\tau$, and the mean geometrical diameter, $D_{hyd}$, of many imaging flow paths to be

$$\frac{d\tau}{dD_{hyd}} = \tau\left(1 - \frac{\tau}{\tau_{max}}\right) \tag{9}$$

with

$$D_{hyd} = d_g(\phi_c^{-\rho_b} - 1) \tag{10}$$

$$\phi_c = 1 - \beta - \frac{\rho_b}{\rho_s} \tag{11}$$

$$\tau_{max} = \left(\frac{\rho_{b0}}{\rho_b}\right)^{\varepsilon} \tag{12}$$

$$\tau_0 = \tau_{max}(1 - \phi_c) \tag{13}$$

In the above equations, $d_g$, $\phi_c$, $\tau_{max}$, and $\tau_0$ are geometric mean particle diameter (mm) calculated by the method of Campbell (1985), saturated flow-active

Table 1
Values of the empirical constants in Eq. (20) used in the study

| Constant | Sand | Loam sand | Sandy loam | Loam | Sandy clay loam | Silty loam | Silty clay | Clay |
|---|---|---|---|---|---|---|---|---|
| $\gamma$ | 0.90 | 0.90 | 0.95 | 0.90 | 0.90 | 0.95 | 0.80 | 0.75 |
| $\omega$ | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 2.0 | 2.0 |

porosity, and the maximum and minimum values of $\tau$, respectively. The parameter, $\beta$, is equivalent to residual water content of soil medium, and can be estimated by the following expression:

$$\beta = 0.015 + 0.005C + 0.014\rho_b \tag{14}$$

where $C$ is the percentage content of clay particle (<2 μm) of soil. The parameter $\varepsilon$ in Eq. (12) is calculated with

$$\varepsilon = \left[\frac{(\rho_s - \rho_b)}{(\rho_{s0} - \rho_{b0})}\right]^{0.5} \tag{15}$$

Table 2
Some parameters of loam soils used in the examination of the models[a]

| Code | Texture | $\rho_b$ (Mg m$^{-3}$) | $K_s$ ($\times 10^{-5}$ cm s$^{-1}$) | $b$ | $N_1$ | $N_2$ | $\tau$ |
|---|---|---|---|---|---|---|---|
| 1380 | SL | 1.38 | 27.8 | 4.39 | 43 | 11 | 0.78 |
| 1381 | SL | 1.73 | 27.7 | 6.67 | 39 | 10 | 0.98 |
| 1390 | SL | 1.64 | 238.0 | 3.91 | 43 | 10 | 0.82 |
| 2532 | SL | 1.33 | 16.7 | 4.76 | 5 | 20 | 0.71 |
| 2541 | SL | 1.70 | 41.9 | 9.30 | 6 | 10 | 0.92 |
| 2551 | SL | 1.85 | 21.0 | 9.04 | 6 | 10 | 0.99 |
| 4160 | SL | 1.70 | 1.2 | 5.41 | 18 | 9 | 0.97 |
| 4162 | SL | 1.52 | 43.6 | 6.10 | 20 | 9 | 0.87 |
| 4170 | SL | 1.46 | 54.7 | 4.24 | 20 | 9 | 0.85 |
| 4172 | SL | 1.59 | 37.4 | 3.88 | 29 | 9 | 0.91 |
| 1370 | L | 0.95 | 68.3 | 8.06 | 45 | 10 | 0.55 |
| 2530 | L | 1.36 | 14.9 | 6.35 | 6 | 20 | 0.73 |
| 2531 | L | 1.46 | 27.0 | 6.57 | 6 | 17 | 0.77 |
| 4101 | L | 1.50 | 10.7 | 9.55 | 25 | 9 | 0.83 |
| 4102 | L | 1.53 | 123.0 | 8.09 | 20 | 9 | 0.84 |
| 4780 | L | 0.49 | 1100.0 | 6.28 | 19 | 17 | 0.35 |
| 4790 | L | 0.96 | 41.0 | 1.80 | 27 | 14 | 0.60 |
| 1382 | SCL | 1.75 | 27.7 | 9.41 | 43 | 10 | 0.96 |
| 2552 | SCL | 1.76 | 38.2 | 4.59 | 6 | 10 | 0.78 |
| 1280 | SiL | 1.35 | 24.2 | 3.45 | 38 | 10 | 0.70 |
| 1490 | SiL | 1.47 | 10.3 | 5.97 | 35 | 10 | 0.74 |
| 2491 | SiL | 1.30 | 5.3 | 8.72 | 8 | 9 | 0.67 |
| 2493 | SiL | 1.43 | 52.7 | 26.70 | 7 | 9 | 0.75 |
| 4030 | SiL | 1.49 | 4.8 | 4.59 | 30 | 9 | 0.78 |
| 4031 | SiL | 1.47 | 6.3 | 4.78 | 24 | 9 | 0.77 |
| 4043 | SiL | 1.53 | 142.0 | 4.60 | 25 | 9 | 0.82 |
| 4100 | SiL | 1.64 | 43.2 | 5.05 | 26 | 9 | 0.88 |
| 4670 | SiL | 1.42 | 103.0 | 3.17 | 25 | 25 | 0.71 |
| 4671 | SiL | 1.56 | 14.2 | 3.96 | 25 | 25 | 0.82 |
| 4673 | SiL | 1.56 | 5.0 | 4.99 | 25 | 25 | 0.81 |

[a] Parameter: $b$ — the fitted parameter in Eq. (5); $N_1$ — number of pairs of $K_{us}(\theta)$–$\psi$; $N_2$ — number of pairs of $\theta$–$\psi$; $\tau$ — the shape factor calculated with Eq. (16). Texture: SL — sandy loam; L — loam; SCL — sandy clay loam; SiL — silty loam.

Thus, the integral form of Eq. (9) is

$$\tau = \left[\frac{\rho_{b0}}{\rho_b}\right]^c \left\{1 + \left[\frac{\rho_s}{\beta\rho_s + \rho_b} - 1\right] \times \exp\left[d_g - d_g\left(1 - \beta - \frac{\rho_b}{\rho_s}\right)^{-\rho_b}\right]\right\}^{-1} \qquad (16)$$

which was used successfully in scaling $K_s$ (Zhuang et al., 2000a). The main assumption embedded in Eq. (8) is that the bulk density and texture of soils can be used to scale $K_s$.

Based on the BC model (Brooks and Corey, 1964), we can write the one-parameter model for predicting $K_{us}(\theta)$ as

$$\frac{K_{us}(\theta)}{K_s} = \rho(\theta)\left(\frac{\theta}{\theta_s}\right)^{2b} \qquad (17)$$

where $\rho(\theta)$ is a correction factor which is assumed to depend on soil water content, bulk density, and texture. Here based on the NSMC embedded in Eq. (8), in unsaturated case we assume

$$\rho(\theta) = \left[\frac{(\tau\rho_s/\rho_{ub})^{1/3} - 1}{(\tau\rho_s/\rho_b)^{1/3} - 1}\right]^2 \qquad (18)$$

where $\rho_{ub}$ is defined as a flow-related bulk density. This starts just from the consideration that there exists different flow-active porosity (Gimenez et al., 1997), or flow-active pore space at different water contents. Therefore, we might define

$$\rho_{ub} = \frac{M}{V_e} \qquad (19)$$

where $M$ is the solid mass per unit volume soil, and $V_e$ is referred to as equivalent unsaturated flow-active pore volume, and assumed to be a function of actual soil bulk volume related to unsaturated water flow. It is defined as

$$V_e = \gamma(V_s + V_w)^{\omega} = \gamma\left(\frac{\rho_b}{\rho_s} + \theta\right)^{\omega} \qquad (20)$$

where $V_s$ and $V_w$ denote volumes of soil solid and liquid phases per unit volume soil, respectively, and $\gamma$ and $\omega$ are two empirical constants related to pore conductivity. They are given in Table 1 for different soil textures. Thus, we have

$$\rho_{ub} = \frac{\rho_b}{[\gamma(\rho_b/\rho_s + \theta)^{\omega}]} \qquad (21)$$

Table 3
Some parameters of sand and clay soils used in the examination of the models[a]

| Code | Texture | $\rho_b$ (Mg m$^{-3}$) | $K_s$ ($\times 10^{-5}$ cm s$^{-1}$) | $b$ | $N_1$ | $N_2$ | $\tau$ |
|---|---|---|---|---|---|---|---|
| 1460 | S | 1.85 | 291.0 | 1.97 | 7 | 10 | 1.00 |
| 1461 | S | 1.65 | 2310 | 2.32 | 7 | 10 | 0.99 |
| 1463 | S | 1.58 | 801.0 | 2.00 | 7 | 10 | 0.98 |
| 1464 | S | 1.67 | 232.0 | 2.02 | 7 | 10 | 0.99 |
| 1467 | S | 1.81 | 13.0 | 3.28 | 5 | 10 | 1.00 |
| 2540 | S | 1.81 | 4.0 | 7.44 | 5 | 10 | 0.99 |
| 2550 | S | 1.57 | 63.0 | 3.82 | 6 | 10 | 0.88 |
| 4000 | S | 1.46 | 111.0 | 3.15 | 11 | 9 | 0.94 |
| 4650 | S | 1.62 | 11.0 | 2.40 | 24 | 25 | 0.99 |
| 4661 | S | 1.49 | 1320.0 | 3.50 | 25 | 25 | 0.93 |
| 4010 | LS | 1.44 | 41.0 | 3.28 | 28 | 11 | 0.89 |
| 4020 | LS | 1.32 | 248.0 | 3.42 | 12 | 9 | 0.83 |
| 1360 | SiC | 1.50 | 2.0 | 11.84 | 33 | 10 | 0.84 |
| 1361 | SiC | 1.49 | 5.4 | 10.34 | 39 | 10 | 0.83 |
| 1383 | SiC | 1.64 | 7.5 | 13.98 | 40 | 10 | 0.87 |
| 3120 | SiC | 1.40 | 4.9 | 4.41 | 10 | 9 | 0.76 |
| 3030 | SiC | 1.32 | 0.5 | 8.95 | 9 | 11 | 0.82 |
| 1400 | C | 1.45 | 0.6 | 14.24 | 27 | 10 | 0.87 |
| 4120 | C | 1.24 | 3110.0 | 14.86 | 22 | 9 | 0.76 |
| 4121 | C | 1.11 | 61.6 | 9.37 | 24 | 9 | 0.72 |
| 4680 | C | 1.10 | 4450.0 | 11.42 | 25 | 25 | 0.72 |
| 4681 | C | 1.08 | 1930.0 | 12.75 | 25 | 25 | 0.74 |

[a] The parameters are the same as those in Table 2. Texture: S — sand; LS — loamy sand; SiC — silty clay; C — clay.

Eventually, unsaturated hydraulic conductivity, $K_{us}(\theta)$, can be computed using

$$K_{us}(\theta) = K_s \left[ \frac{(\tau\rho_s/\rho_{ub})^{1/3} - 1}{(\tau\rho_s/\rho_b)^{1/3} - 1} \right]^2 \left( \frac{\theta}{\theta_s} \right)^{2b} \quad (22)$$

where τ was estimated using Eq. (16) by simply taking $\rho_{b0} = \rho_b$. We term Eq. (22) as NSMC-based model of $K_{us}(\theta)$ (hereafter called NSMC model).

## 3. Data sets and statistical evaluation

For comparison of the different models in predicting $K_{us}(\theta)$ of soils, data sets, including data of $K_s$, measured curves of water retention characteristic and unsaturated hydraulic conductivity, fractions of sand, silt and clay, bulk density and particle density of 52 soils with textures ranging from sand to clay, were selected from the UNSODA hydraulic property database (Nemes et al., 1999), and used in the study. Tables 2 and 3 give the soil codes and some parameters of the database. For the data sets in which data of particle density are not included, constant of 2.65 Mg $m^{-3}$ was used. For the data sets that do not include data of saturated water content, 95% of the total porosity of soil was assumed as the data of this parameter and employed in the study. In the simulation, in order to reduce the estimation errors arising from air-entry potential and to increase estimation accuracy for $b$ fitted by Eq. (5), soil water retention data for $\psi > -30$ cm $H_2O$ were excluded when fitting the values of $b$. Moreover, to make quantitative comparison of the models, statistical

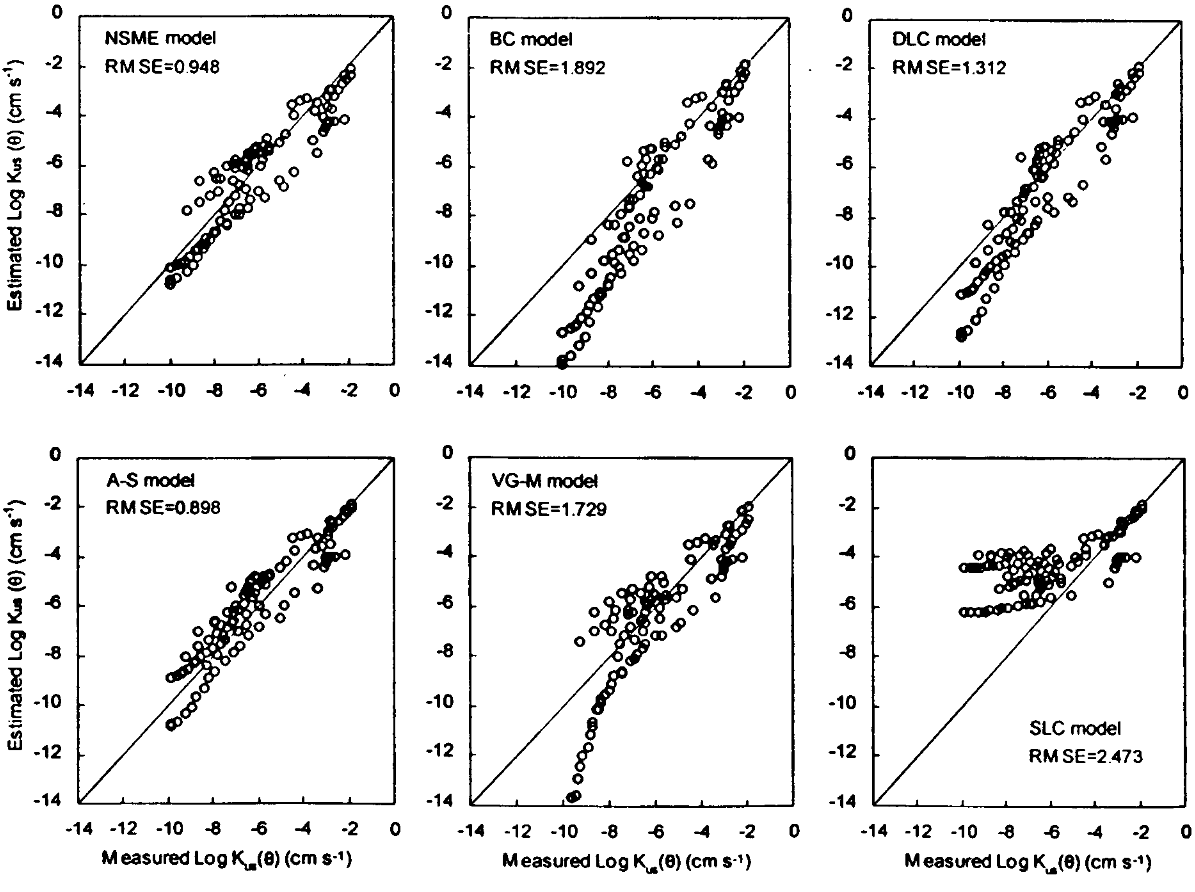

Fig. 1. Comparison of $K_{us}$ (θ) measured and estimated by means of the six models for sand soils. The number of pairs of data is 104.

parameter, RMSE (root mean square error), was used as an assessing standard, which is calculated with

$$\mathrm{RMSE} = \left[\left(\frac{1}{N}\right)\sum_{i=1}^{N}(\log(K_{\mathrm{measured}}) - \log(K_{\mathrm{estimated}}))^2\right]^{0.5} \tag{17}$$

where $K_{\mathrm{measured}}$ and $K_{\mathrm{estimated}}$ represent unsaturated hydraulic conductivity measured and estimated by means of the models, respectively, and $N$ indicates number of hydraulic conductivity data. Smaller value of RMSE indicates smaller deviation, or higher agreement between the values estimated and measured.

## 4. Results and discussion

By using 1093 pairs of $K_{\mathrm{us}}(\theta)$–$\theta$ data of 52 soil samples with textures ranging from sand to clay, the models were examined. For all soil textures, the NSMC model had the smallest RMSE, i.e. 1.209 log cm s$^{-1}$, indicating its higher accuracy in predicting $K_{\mathrm{us}}(\theta)$ as compared to the other models. The next two better models are the BC model and the DLC model, with the value of RMSE being 1.522 log cm s$^{-1}$, and 1.621 log cm s$^{-1}$, respectively. The A–S model ranked fourth, with the value of RMSE being 1.947 log cm s$^{-1}$. In contrast, two models having larger deviations in predicting $K_{\mathrm{us}}(\theta)$ are the VG–M model and the SLC model. For the former, the value of RMSE is 2.088 log cm s$^{-1}$, and for the latter

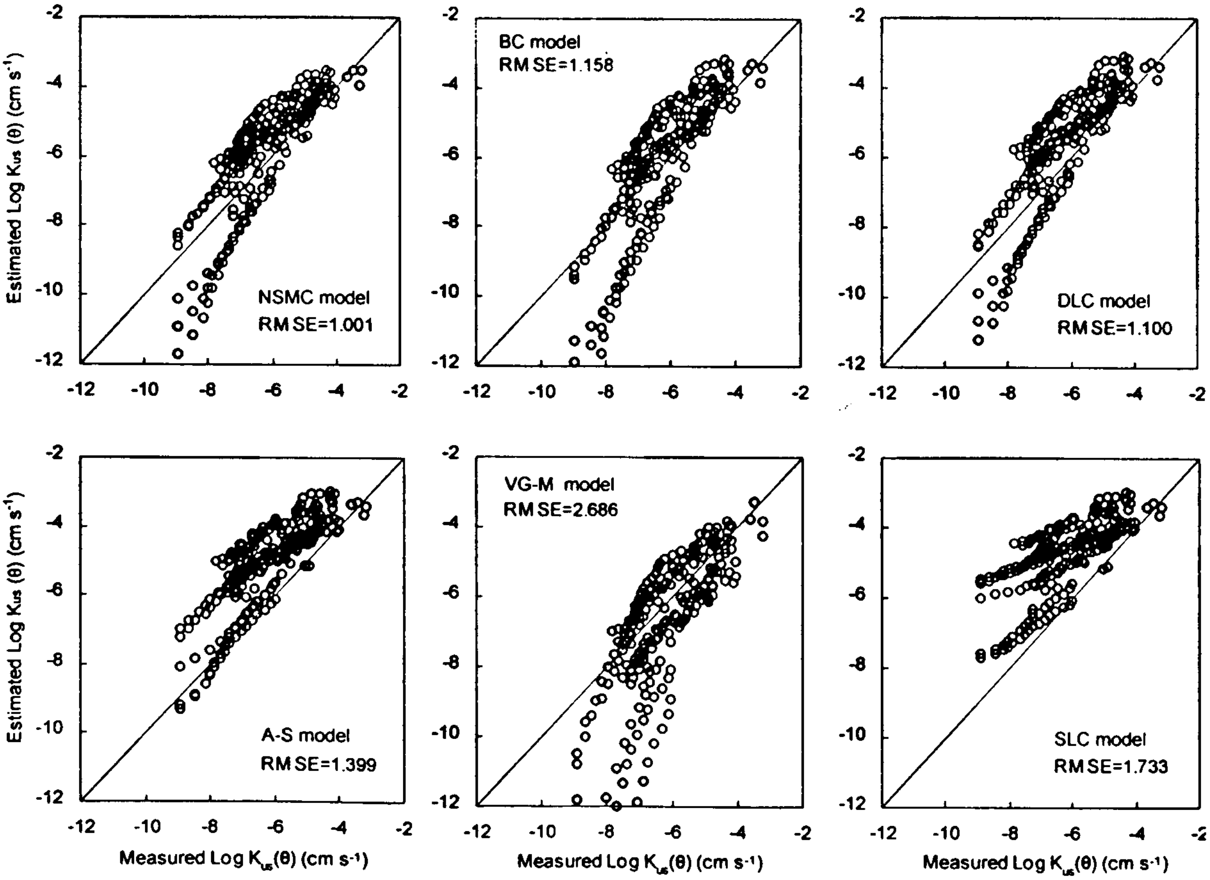

Fig. 2. Comparison of $K_{\mathrm{us}}$ $(\theta)$ measured and estimated by means of the six models for sandy loam soils. The number of pairs of data is 247.

2.366 log cm s$^{-1}$. As we know that RMSE is 1 log cm s$^{-1}$ means that the estimated values, on average, deviate 10 times from the measured values, and 2 log cm s$^{-1}$ is equivalent to 100 times deviation. Thus, it could be said that all of the six models have limited ability of predicting $K_{us}(\theta)$. In comparison, the NSMC, BC and DLC models are more reliable if $K_{us}(\theta)$ of soils including various textures are predicted using data of water retention characteristics.

With regard to applicability of the models to individual soil textures, Figs. 1–6 compare the six models, and show that accuracy of the prediction with each model differed for different textural groups. For sand soils (Fig. 1), the A–S model is shown to be the best one, a little superior to the NSMC model in estimating $K_{us}(\theta)$. Also, Fig. 1 shows that the DLC model is a better model for estimating $K_{us}(\theta)$ of sand soils. For sandy loam soils (Fig. 2), it is easily seen that the NSMC, DLC and BC models have higher accuracy of predicting $K_{us}(\theta)$ than the other three models. For loam soils (Fig. 3), the prediction accuracy of the six models are lower than that for sand and sandy loam soils. In comparison, the NSMC, BC and DLC models behaved better. For silty loam soils (Fig. 4), the estimation deviations of most of the models are smaller than those for loam soils, and generally similar to those of sandy loam soils. For silty clay soils, Fig. 5 shows that except for the NSMC and BC models, all of the other models have larger deviations in estimating $K_{us}(\theta)$. Moreover, from Fig. 6, it is obvious that large estimation deviations were made by all the models for clay soils, as compared to loam and sand soils. To sum up, if we categorize soil textures into only three groups, such as, sand, loam and clay, the NSMC model

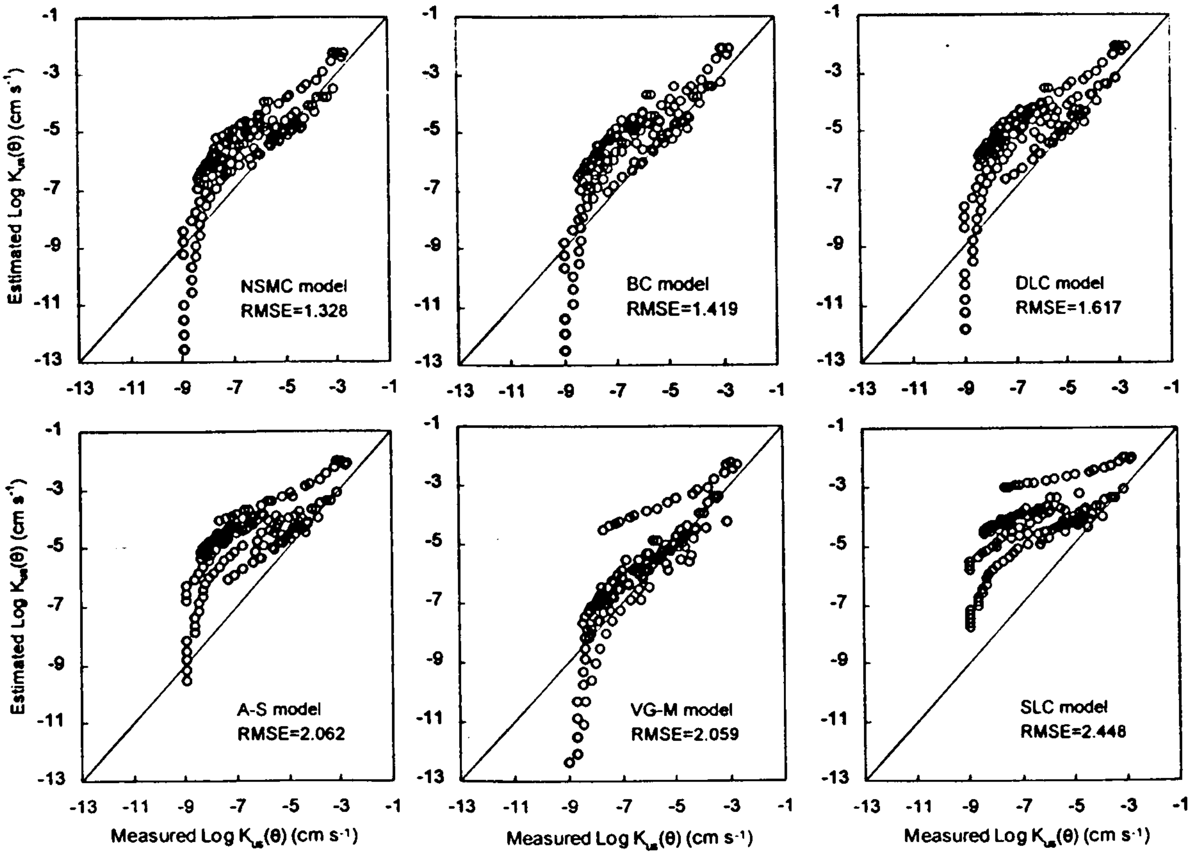

Fig. 3. Comparison of log $K_{us}$ ($\theta$) measured and estimated by means of the six models for loam soils. The number of pairs of data is 148.

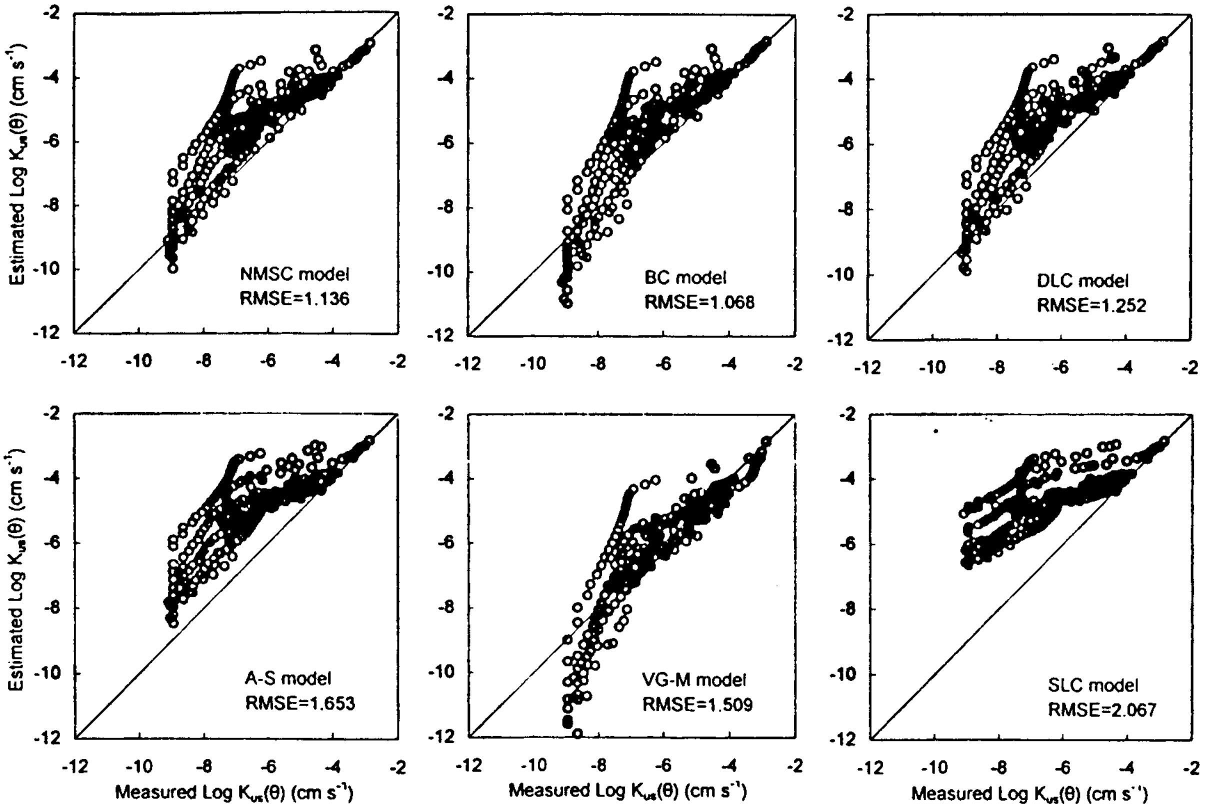

Fig. 4. Comparison of $K_{us}(\theta)$ measured and estimated by means of the six models for silty loam soils. The number of pairs of data is 268.

holds true for sand and loam soils. The BC model is applicable to loam soils. The DLC model generally has a smaller estimation deviation for sand and loam soils. The A–S model seems to be more suitable to estimate $K_{us}(\theta)$ of sand soil or loam soil with more sand particles, like sandy loam soil. The VG–M model had a better estimation of $K_{us}(\theta)$ for silty loam soil than for sand and clay soils. For the SLC model, unfortunately, large estimation deviations were gotten for soils of all textures.

Generally, estimation errors of hydraulic conductivity may be induced both by an inappropriate fitting of the soil water retention and by failure of the estimation models of hydraulic conductivity. For the one-parameter model, the estimation errors perhaps arose from the assumption that relative hydraulic conductivity, $K_{us}(\theta)/K_s$, depends solely on the soil water retention characteristic, while actually other factors, such as bulk density, specific surface area of soil, also exert some effects on it. Moreover, another possible source of the estimation errors is the inflexibility of Brooks and Corey's soil-water retention model. For the VG–M model, although the water retention model of van Genuchten (1980) is very flexible, the assumption underlying the statistical integral model of relative hydraulic conductivity (Mualem, 1976) restricts its flexibility seriously. For NSMC model, its superiority to other models in predicting $K_{us}(\theta)$ is presumably ascribed to the incorporation of bulk density and parameters related to soil texture, e.g., $d_g$, into the model through the shape factor $\tau$ defined in the NSMC theory and $V_e$, the equivalent unsaturated flow-active pore volume defined in Eq. (19). This could be easily comprehended because coupled effects of bulk density and texture usually play an important role in determining pore size, shape, and orientation and significantly influence hydraulic conductivity (Bouma, 1992).

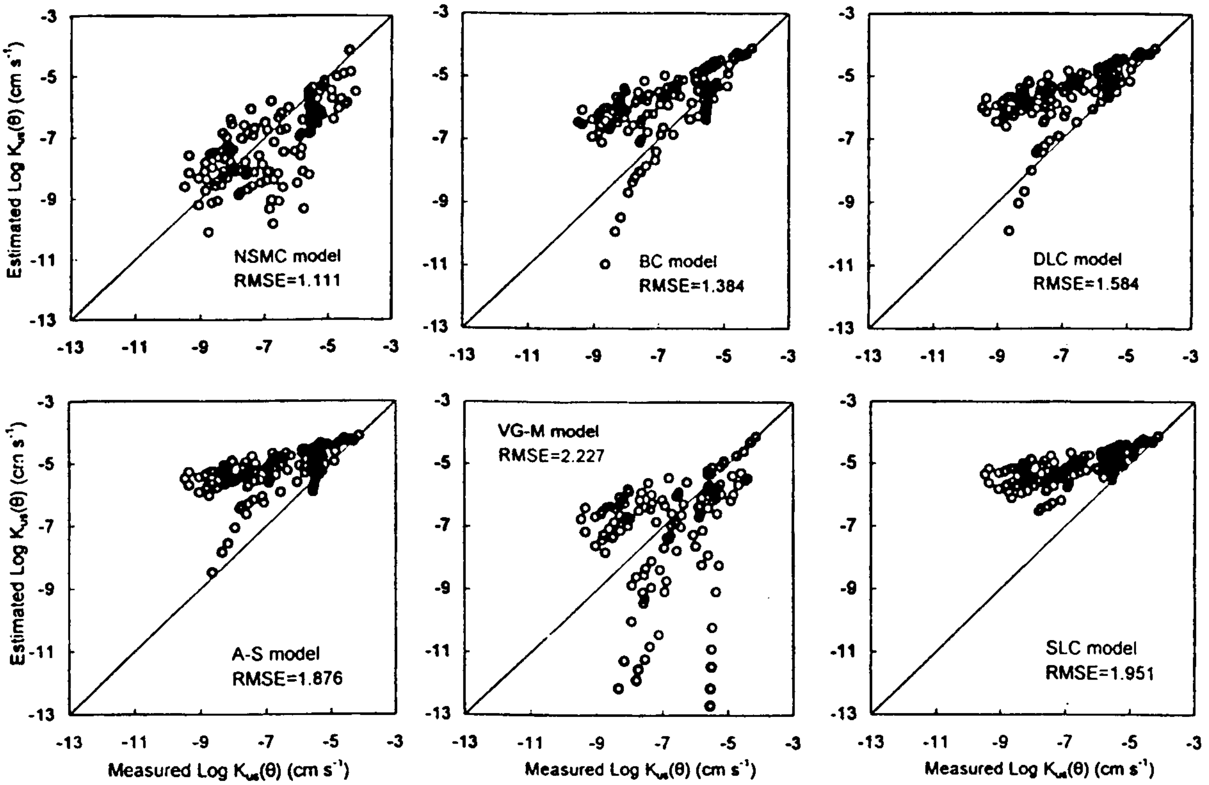

Fig. 5. Comparison of $K_{us}(\theta)$ measured and estimated by means of the six models for silty clay soils. The number of pairs of data is 132.

Therefore, perhaps, it is not wrong to assume that water retention characteristic could not completely and precisely represent assemblage characteristic of soil particles, or that it is unreasonable to determine soil hydraulic conductivity solely depending on the distribution of the equivalent pore represented by water retention curve. Note that hydraulic conductivity is an intensity property, while water retention is a capacity property. Pore connectivity usually substantially impacts hydraulic flow. A negligible estimation deviation for the water retention characteristic may induce a significantly large deviation in the estimation of $K_{us}(\theta)$. Therefore, new theories or concepts that mechanically include component of pore/particle arrangements are urgently needed in the simulation of soil flow. Those models formulated with information of arrangement of soil particles should be recommended to use in the modeling practices of soil hydraulics, or to give preferential improvements in the future.

## 5. Conclusions

A new model for predicting absolute $K_{us}(\theta)$ or relative hydraulic conductivity, $K_{us}(\theta)/K_s$, was proposed by combining NSMC to the one-parameter model of Brooks and Corey. The developed model was compared with other five widely used models using hydraulic data of 52 soils. Although none of the tested models had a consistently better prediction for all textural classes, the NSMC model generally performed best, having a tendency of being more suitable to the sand and loam soils. The second best model was the BC model, while the DLC model presented by Poulsen et al. (1998) ranked third. The A–S model generally performed fourthly better, while the VG–M model ranked fifth in terms of estimation deviation represented by RMSE of log $K_{us}(\theta)$. The model comparison performed in this study emphasized the critical importance of incorporating soil bulk density and soil texture into the models of deriving soil hydraulic

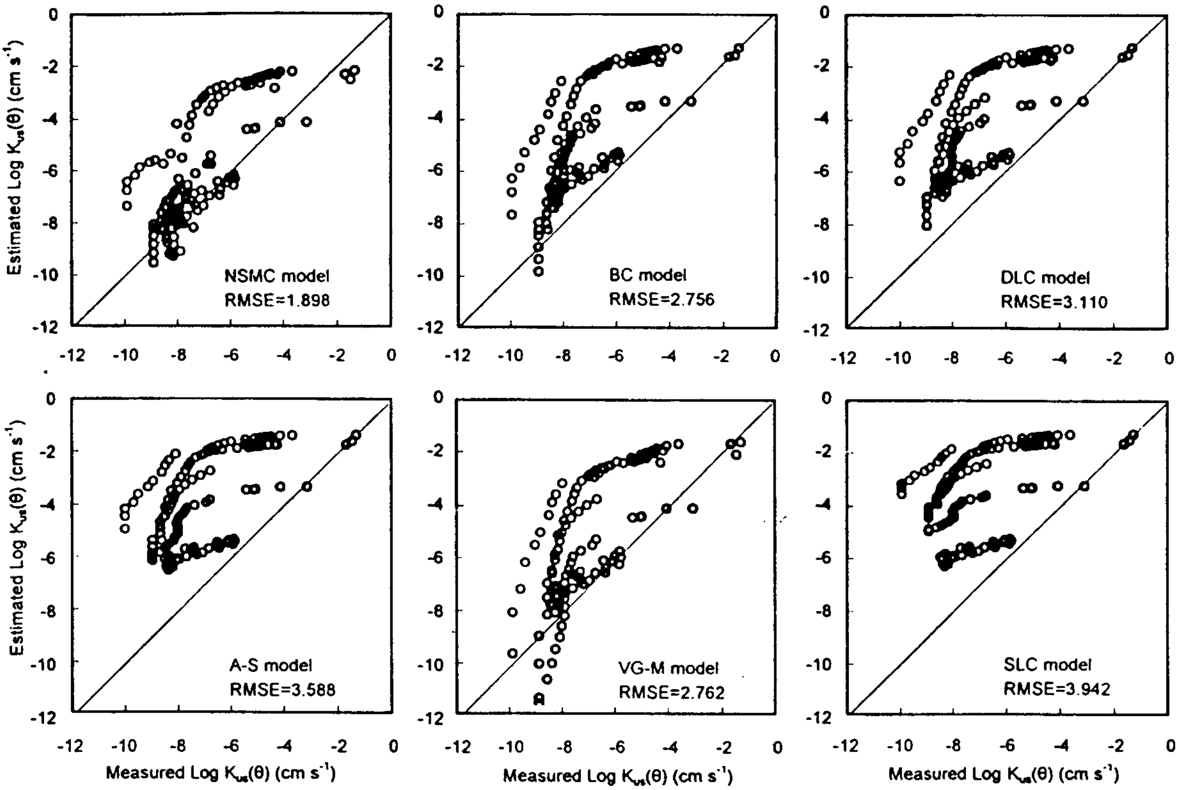

Fig. 6. Comparison of $K_{us}$ ($\theta$) measured and estimated by means of the six models for clayey soils. The number of pairs of data is 123.

conductivity from soil water retention characteristics. However, the new model still needs to be improved by considering effects of soil structure and soil internal architecture on hydraulic properties.

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